The heat equation

1. Suppose the coefficient is $\alpha = 1.25$ for a system that is initially at ambient temperature (20˚C) and one end of a connector is maintained at 20˚C through air cooling, while the other is in contact with a processor that heats up to 80˚C. Suppose the connector is 5 mm wide and you'd like to simulate heat flow through the connector, so you divide the connector into 1 mm wide sub-intervals.

a. The $\alpha = 1.25$ has units proportional to per meter, so what should the unit be if we are working on the millimeter scale?

Answer: 1250.0

b. What is an appropriate value of Δt?

Answer: $\Delta t = 0.0002$

c. What is the approximation of the values at $t = 0.0002$ and $t = 0.0004$?

Answer: Note that $h = 1$, so plugging in the values yield the numbers given, for example,

$$20 + 0.0002 \times 1250 \times (20 - 2\times 20 + 80)/1^2 = 35$$

80	80	80
20	35	42.5
20	20	23.75
20	20	20
20	20	20
20	20	20

d. Once you have completed the topic on Laplace's equation, what the temperature distribution tend towards as t approaches infinity?

Answer: 80 68 56 44 32 20

2. If we halve α, that is, introduce a more insulating material, what will happen to the propagation of heat?

Answer: It will be slower, at propagating through the material. For example,

$$20 + 0.0002 \times 625 \times (20 - 2\times 20 + 80)/1^2 = 27.5$$

80	80	80
20	27.5	33.125
20	20	20.9375
20	20	20
20	20	20
20	20	20

Acknowledgement: Martin Szlapa for noting two digits were transposed in the solution to Question 1c.